

# Interference pattern of the Coulomb and the strong Van der Waals forces in p-p scattering\*

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## Abstract

In order to confirm the strong Van der Waals force in the nucleon-nucleon interaction, it is proposed to measure precisely the angular distribution of the cross section of the low energy ( $T_{lab} = 20 \sim 30\text{MeV.}$ ) proton-proton scattering. By using the spectrum of the long range interaction obtained from the analysis of the phase shift data of the S-wave of the p-p scattering, a characteristic interference pattern, which arises from the repulsive Coulomb and the attractive strong Van der Waals forces, is predicted. The pattern has a dip at  $\theta_{c.m.} = 14^\circ$  with the depth around one per cent.

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# 1 Introduction

In the composite model of hadron,[1] in which the fundamental constructive force is a strong or super-strong Coulombic force, the quantum fluctuation of the composite states causes the strong Van der Waals interaction between hadrons. Historically before 1960's, hadrons were regarded as elementary particles and the interactions arose from the exchange of mesons with masses, therefore the forces were inevitably short range. After the introduction of the composite model of hadron, the idea of the short range force is taken over to the new hadron physics, because when the momentum transfer is small and we do not explore the inside of hadrons, there must be no differences whether the hadrons are composite or elementary. Although the short range nature of the interactions between hadrons are widely believed, it cannot be true when the strong Van der Waals forces are acting between hadrons.[2] The purpose of the present paper is to propose an experiment to confirm the existence of the strong Van der Waals interaction between nucleons by observing the characteristic interference in the low energy ( $20 \sim 30$  MeV.) proton-proton scattering.[3] The interference pattern arises from the destructive interference between the repulsive Coulomb and the attractive strong Van der Waals forces, and the cross section  $\Delta\sigma/\sigma$  has a narrow dip in the neighborhood of  $\theta_{c.m.} = 14^\circ$  with the depth around one per cent.

In the confirmation of the strong Van der Waals interaction, it is desirable to have recourse to the difference of analytic structure of the scattering amplitude  $A(s, t)$  in the neighborhood of  $t = 0$ . For the case of the short range force  $A(s, t)$  is regular at  $t = 0$  and the nearest singularity occurs at  $t = t_{min}$ , where  $(t_{min})^{1/2}$  is the smallest mass exchanged in the t-channel. On the other hand when the long range force is acting and the asymptotic form of the potential is  $V(r) \sim -C/r^\alpha$ , an extra singularity occurs at  $t = 0$  in  $A(s, t)$  and whose spectral function behaves as  $A_t(s, t) = C't^\gamma + \dots$  in the threshold region. In the next section, we shall see that the powers  $\alpha$  and  $\gamma$  are related by  $\alpha = 2\gamma + 3$ . Therefore to observe the extra singularity at  $t = 0$  implies the confirmation of the long range force. Moreover since  $t = 0$  is the end point of the physical region  $-4\nu \leq t \leq 0$ , we can recognize the extra singularity, when the sufficiently precise data are available, without making any analytic continuation.

In particular if the long range interaction is the Van der Waals potential of the London type ( $\alpha = 6$ ), the amplitude  $A(s, t)$  must have a singular term  $C'(-t)^{3/2}$ . Since  $-t = 2\nu(1 - z)$ , there are two ways to observe the extra singularity of the amplitude  $(2\nu)^{3/2}(1 - z)^{3/2}$ . The first one is to make the partial wave projection and to observe the singular threshold behavior  $\nu^{3/2}$  in the partial wave amplitudes  $a_\ell(\nu)$ . The second one is to fix  $\nu$  and to observe the anomalous angular dependency  $(1 - z)^{3/2}$ , which has a singularity at  $z = 1$ . By analysing the once subtracted S-wave amplitude of the p-p scattering  $(a_0(\nu) - a_0(0))/\nu$ , the long range force was searched in the previous paper,[4] and we observed a cusp of the form  $(c_0 - c_1\sqrt{\nu})$  at  $\nu = 0$  which was characteristic to the Van der Waals interaction of the London type. In section 2, we shall briefly review the previous search of the strong Van der Waals force using the once subtracted partial wave amplitude, and the parameters of the long range interaction will be given. In section 3, by using the parameters of the long range force, the anomalous angular

distribution of the amplitude of the p-p scattering is computed, and the characteristic interference pattern of the cross section is predicted. Section 4 will be used for remarks and comments.

## 2 Search for the strong Van der Waals force in the S-wave amplitude

When the scattering amplitude  $A(s, t)$  has the extra singularity  $C'(-t)^\gamma$ , the partial wave amplitudes  $a_\ell(\nu)$  also have extra singularities at  $\nu = 0$  and the threshold behaviors become  $C''_\ell \nu^\gamma$ , where  $C''_\ell$  are proportional to  $C'$ . Since in the hadron physics the data of the S-wave phase shift of the p-p scattering are prominent in their accuracy and the scattering length is determined very precisely, we shall analyse the once subtracted S-wave amplitude  $(a_0(\nu) - a_0(0))/\nu$  in search for the extra singularity  $C''_0 \nu^{\gamma-1}$ . Our normalization of the partial wave amplitude  $a_0(\nu)$  is

$$a_0(\nu) = \frac{\sqrt{m^2 + \nu}}{X_0(\nu) - i\sqrt{\nu}} \quad (1)$$

for the scattering of neutral particles, and  $X_0(\nu)$  is the effective range function  $\sqrt{\nu} \cot \delta_0(\nu)$  which is regular at  $\nu = 0$  and accepts the Taylor expansion of  $\nu$ . In order to observe the singular behavior at  $\nu = 0$ , we must remove the known nearby singularities. First of all we have to remove the unitarity cut by constructing a function

$$\frac{K_0(\nu)}{\nu} \equiv \frac{a_0(\nu) - a_0(0)}{\nu} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im} a_0(\nu')}{\nu'(\nu' - \nu)} d\nu' \quad (2)$$

, which is free from the right hand cut and is called the Kantor amplitude.[5] To facilitate the search of the extra singularity it is desirable to remove also the cut of the one-pion exchange (OPE). The procedure is the same as the case of the unitarity cut, namely first compute an integration

$$\frac{K_0^{1\pi}(\nu)}{\nu} \equiv \frac{1}{\pi} \int_{-\infty}^{-1/4} \frac{\text{Im} a_0^{1\pi}(\nu')}{\nu'(\nu' - \nu)} d\nu' = \frac{1}{4} \frac{g^2}{4\pi} \left\{ \frac{1}{4\nu} \log(1 + 4\nu) - 1 \right\} \quad (3)$$

, and then subtract it from the amplitude. In the calculation, the neutral pion mass is set equal to 1, and throughout this paper we shall use the neutral pion mass as the unit of the energy and the momentum. Since the two-pion exchange spectrum starts slowly at  $\nu = -1$ , the function  $(K_0(\nu) - K_0^{1\pi}(\nu))/\nu$  must be almost constant and have small slope in the neighborhood of  $\nu = 0$  when the long range interactions are absent. On the other hand, for the Van der Waals interaction of the London type ( $\alpha = 6$ )  $\gamma = 3/2$  and  $(K_0(\nu) - K_0^{1\pi}(\nu))/\nu$  must have a singular term  $C'' \nu^{1/2}$ , therefore we can observe a cusp at  $\nu = 0$  as long as the coefficient  $C''$  is not very small. In this way we can examine the long range force if the  $\pi$ -N coupling constant  $g^2/4\pi$  and the S-wave phase shifts are given. Because the effects of the Coulomb and the vacuum polarization potentials are not considered, this method is applicable to the case where the Coulomb interaction is not important such as to the neutron-neutron scattering.

Let us turn to the proton-proton scattering, where the vacuum polarization as well as the Coulombic interactions are important. The Kantor amplitude introduced in Eq.(2) still has the left hand cuts, namely the Coulombic cut in  $-\infty < \nu \leq 0$  and the cut of the vacuum polarization in  $-\infty < \nu \leq -m_e^2$ . The difficulties are by-passed if we use the modified effective range function  $X_0(\nu)$  of the proton-proton scattering, which is regular at  $\nu = 0$  and accepts the effective range expansion, when all the forces are short range except for the terms of the Coulomb and of the vacuum polarization. The modified effective range function  $X_0(\nu)$  for the phase shift  $\delta_0^E(\nu)$  is

$$X_0(\nu) = \frac{C_0^2 \sqrt{\nu}}{1 - \phi_0} \{ (1 + \chi_0) \cot \delta_0^E - \tan \tau_0 \} + me^2 h(\eta) + me^2 \ell_0(\eta) \quad . \quad (4)$$

In Eq.(3), two well-known functions with the Coulombic order of magnitudes appear, they are expressed using a new variable  $\eta = me^2/(2\sqrt{\nu})$  :

$$C_0^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad \text{and} \quad h(\eta) = \eta^2 \sum_{\ell=1}^{\infty} \frac{1}{\ell(\ell^2 + \eta^2)} - \log \eta - 0.57722 \dots \quad (5)$$

In Eq.(4)  $\tau_0$  is the phase shift due to the vacuum polarization potential[6]

$$V^{vac}(r) = \lambda \frac{e^2}{r} \int_{4m_e^2}^{\infty} dt \frac{e^{-r\sqrt{t}}}{2t} \left(1 + \frac{2m_e^2}{t}\right) \sqrt{1 - \frac{4m_e^2}{t}} \equiv \lambda \frac{e^2}{r} I(r) \quad , \quad (6)$$

where  $m_e$  is the mass of the electron and  $\lambda = 2e^2/3\pi = 1.549 \times 10^{-3}$ . Functions  $\tau_0$ ,  $\chi_0$ ,  $\phi_0$  and  $\ell_0(\eta)$  have the order of magnitudes of the vacuum polarization, and introduced in the previous paper.

By using the modified effective range function  $X_0(\nu)$ , we define the S-wave amplitude  $a_0(\nu)$  of the p-p scattering by

$$a_0(\nu) = \frac{\sqrt{m^2 + \nu}}{X_0(\nu) - me^2 h(\eta) - i\sqrt{\nu} C_0^2} \quad . \quad (7)$$

The relation between  $a_0(\nu)$  and the phase shift  $\delta_0^E$  is obtained if we substitute  $X_0(\nu)$  of Eq.(4) into Eq.(7), and which reduces to the well-known form

$$a_0(\nu) = \frac{1}{C_0^2} \frac{\sqrt{m^2 + \nu}}{\sqrt{\nu}} e^{i\delta_0^E(\nu)} \sin \delta_0^E(\nu) \quad , \quad (8)$$

if the functions related to the vacuum polarization are neglected. The form of  $a_0(\nu)$  of Eq.(7) is the same as that of the neutron-neutron scattering  $(\sqrt{m^2 + \nu}/\sqrt{\nu}) e^{i\delta} \sin \delta$  except for the factor  $C_0^2$  given in Eq.(4), which is the penetration factor. If we compare the S-wave amplitude of the p-p scattering of Eq.(7) with that of the n-n scattering Eq.(1), a combination of functions  $(-me^2 h(\eta) - i\sqrt{\nu} C_0^2)$  appears in place of  $-i\sqrt{\nu}$ . In order to investigate the analytic structure of  $a_0(\nu)$ , it is convenient to rewrite the combination as

$$-me^2 h(\eta) - i\sqrt{\nu} C_0^2 = -i\sqrt{\nu} + me^2 \{ \log(i\eta) - \psi(1 + i\eta) \} \quad . \quad (9)$$

Since the digamma function  $\psi(z)$  has poles at non-positive integers, the poles on the  $\eta$ -plane appear on the positive imaginary axis. In terms of  $\sqrt{\nu}$ , which is  $me^2/(2\eta)$ , the series of poles appear on the negative imaginary axis and converge to  $\sqrt{\nu} = 0$ . It is the smallness of the fine structure constant  $e^2$  and therefore of the residues of such poles that zeros of the denominator of Eq.(7) occur at points very close to the locations of the poles of  $(-me^2h(\eta) - i\sqrt{\nu}C_0^2)$ . Therefore the partial wave amplitude  $a_0(\nu)$  of the p-p scattering has a series of poles on the second sheet of  $\nu$ , namely on the lower half plane of  $\sqrt{\nu}$ , whereas on the first sheet of  $\nu$  the analytic structure of  $a_0(\nu)$  does not change compared to the case of the n-n scattering. This fact implies that the same definition of the Kantor amplitude  $K_0(\nu)$  introduced for the neutron-neutron scattering, which is given in Eq.(2), is valid also for the proton-proton scattering, as long as we evaluate  $\text{Im}a_0(\nu')$  of Eq.(2) from Eqs.(7) and (9). Therefore the Kantor amplitude of the p-p scattering  $K_0(\nu)$  constructed in this way is free from the singularities in the neighborhood of  $\nu = 0$ , and so does not have the cut of the vacuum polarization as well as that of the Coulomb interaction.

We can now compute the once subtracted S-wave Kantor amplitude minus the contribution from the one-pion exchange of the proton-proton scattering:

$$\tilde{K}_0^{once}(\nu) \equiv \frac{K_0(\nu)}{\nu} - \frac{K_0^{1\pi}(\nu)}{\nu} . \quad (10)$$

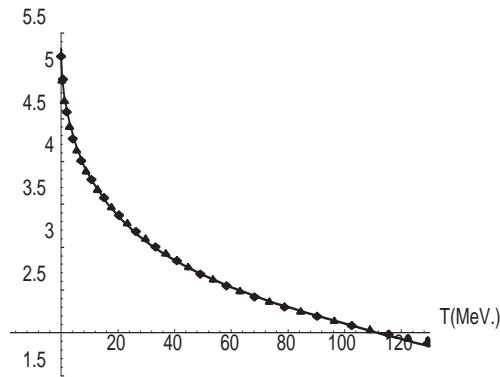


Figure 1:  $-\tilde{K}_0^{once}(\nu)$  is plotted against  $T_{lab}$  in  $T_{lab} < 125 \text{ MeV.}$ . The curve is the fit by the spectrum of the long range force with three parameters in the energy range  $0.6 \text{ MeV.} < T_{lab} < 125 \text{ MeV.}$  .

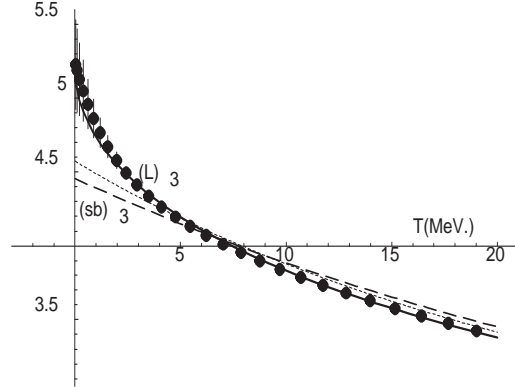


Figure 2: The enlarged graph of  $-\tilde{K}_0^{once}(\nu)$  is plotted in  $T_{lab} < 20 \text{ MeV.}$ . The three curves are 3-parameter fits to the data in  $0.6 \text{ MeV.} < T_{lab} < 125 \text{ MeV.}$ . The dotted and the dashed curves are the fits by the spectra of the short range forces.

In figure 1 and figure 2,  $-\tilde{K}_0^{once}(\nu)$  is plotted against  $T_{lab}$ . The graphs exhibit a cusp at  $\nu = 0$ , which is characteristic to the attractive long range force. The cusp is fitted by a spectral function of three parameters:

$$A_t^{extra}(s, t) = \pi C' t^\gamma e^{-\beta t} \quad (11)$$

, and the results of the chi-square fit in  $0.6\text{MeV} < T_{lab} < 125\text{MeV}$ . are

$$\gamma = 1.543 \quad , \quad \beta = 0.06264 \quad \text{and} \quad C' = 0.1762 \quad (12)$$

in the unit of the neutral pion Compton wave length. The curve in fig.1 and  $(L)_3$  curve in fig.2 are the fits by the spectral function of the long range force  $A_t^{extra}(s, t)$  given in Eqs.(11) and (12), and the  $\chi$ -value per data point is 0.441. On the other hand, other curves in fig.2 are the three parameter fits by the spectral function of the short range forces:

$$A_t(s, t) = \sum_{i=1}^3 c_i \delta(t - t_i) \quad , \quad (13)$$

where  $c_i$  are free parameters and three  $t_i$  are 4, 9 and 16 for the curve  $(sa)_3$  (dotted curve), whereas  $t_i$  are 9, 16 and 25 for the curve  $(sb)_3$  (dashed curve), and the  $\chi$ -values of the fits per data points are 1.82 and 3.11 respectively. The curves indicate that the short range spectra, which mimic the spectrum of the two-pion exchange, cannot reproduce the cusp of  $-\tilde{K}_0^{once}(\nu)$  shown in figures 1 and 2. Details of the fits are found in the previous paper.[4]

### 3 Interference pattern of the cross section of p-p scattering

In the previous section, we observed a cusp at  $\nu = 0$  in  $-\tilde{K}_0^{once}(\nu)$  which arises from the partial wave projection of the singular term  $C'(2\nu)^\gamma(1-z)^\gamma$ . However the same singularity can also be observed in the angular distribution for fixed  $\nu$ . The aim of this paper is to propose to observe the singularity at  $z = \pm 1$  and to confirm the existence of the strong Van der Waals force in the p-p scattering. Since the Coulomb potential also gives rise to the poles at  $z = \pm 1$ , we can expect to observe the characteristic interference pattern of the singular behaviors in the cross section. In this section, we shall compute such a pattern by using the parameters of the long range force determined in the previous section. If we consider that the energy dependent phase shift data are extracted from the measurements of different laboratories, the observation of the singular behavior in the angular distribution is the more direct way to confirm the existence of the long range force.

The amplitude due to the extra spectrum of Eq.(11) is

$$\begin{aligned} \frac{m}{\sqrt{\nu}} W(\nu, z) &= \frac{\pi C'}{\pi} \int_0^\infty dt' \frac{t'^\gamma e^{-\beta t'}}{t' + 2(1-z)} \\ &= C'(2\nu)^\gamma(1-z)^\gamma \Gamma(\gamma+1) \Gamma(-\gamma, 2\nu\beta(1-z), \infty) \exp[2\nu\beta(1-z)] \end{aligned} \quad (14)$$

, where  $\Gamma(x, a, b)$  is the incomplete gamma function defined by  $\int_a^b t^{x-1} e^{-t} dt$ . The cross section  $\sigma(\theta)$  of the proton-proton scattering in the low energy region is[3]

$$\begin{aligned} \nu \sigma(\theta) &= \frac{1}{4} |\tilde{f}^s(\theta) + 2 \exp[i\delta_0^E] \sin \delta_0^E + \alpha_e(z)|^2 + \\ &+ \frac{3}{4} |\tilde{f}^t(\theta) + 6\delta_{1,C} z + \alpha_o(z)|^2 + \Delta \quad . \end{aligned} \quad (15)$$

$\tilde{f}^s(\theta)$  and  $\tilde{f}^t(\theta)$  are the Coulomb amplitude plus the one-pion exchange contribution of the spin-singlet and the spin-triplet states respectively, and they are

$$\begin{aligned}\tilde{f}^s(\theta) &= -\frac{\eta}{2}\left\{\frac{1}{S^2}\exp[-i\eta\log S^2] + \frac{1}{C^2}\exp[-i\eta\log C^2]\right\} + \\ &+ f^2m\sqrt{\nu}\left\{\left(\frac{1}{1-t} + \frac{1}{1-u}\right) - \frac{2}{2\nu}Q_0\left(1 + \frac{1}{2\nu}\right)\right\}\end{aligned}\quad (16)$$

and

$$\begin{aligned}\tilde{f}^t(\theta) &= -\frac{\eta}{2}\left\{\frac{1}{S^2}\exp[-i\eta\log S^2] - \frac{1}{C^2}\exp[-i\eta\log C^2]\right\} - \\ &- \frac{f^2}{3}m\sqrt{\nu}\left\{\left(\frac{1}{1-t} + \frac{1}{1-u}\right) - \frac{6}{2\nu}Q_1\left(1 + \frac{1}{2\nu}\right)z\right\},\end{aligned}\quad (17)$$

, in which

$$S = \sin \frac{\theta}{2}, \quad C = \cos \frac{\theta}{2} \quad \text{and} \quad \eta = \frac{e^2}{\hbar c \beta_L}, \quad (18)$$

where  $\beta_L$  is the velocity in the laboratory system and it is written in terms of  $\nu$

$$\beta_L = \frac{2\sqrt{\nu}\sqrt{1 + \frac{\nu}{m^2}}}{m(1 + \frac{2\nu}{m^2})}. \quad (19)$$

In Eq.(15)  $\alpha_e(z)$  and  $\alpha_o(z)$  are the even and odd functions of  $z$  which take care of the contributions from the higher partial waves and for very small  $\nu$  they are negligible. When the hadron interaction is short range, since the poles of the one-pion exchanges are already separated, the nearest singularity of  $\alpha(z)$ 's on the  $z$ -plane occurs at

$$z = \pm\left(1 + \frac{t_{min}}{2\nu}\right) \quad (20)$$

with  $t_{min} = 4$ , the threshold of the spectrum of the two-pion exchange. As an example, we shall fix the incident energy at  $T_{lab} = 20$  MeV., namely at  $\nu = 0.515$ . In such a case, the analytic domain on the  $z$ -plane is the Lehmann ellipse whose semi-major axis is around 5 and foci are at  $z = \pm 1$ . However if the two-pion exchange spectrum is very small in the threshold region and the main cotribution comes from the  $\sigma$ -meson (560 MeV.), then  $t_{min} = 16$  and the semi-major axis is around 17. For  $\alpha_e(z)$  it is sufficient to retain only the S and D waves to reproduce the amplitude within the error  $10^{-3}$  in the physical region  $-1 \leq z \leq 1$ . For the odd function  $\alpha_o(z)$  we shall consider two cases, the first one is to retain only the P wave, whereas the second one is to retain the P and F waves.

To observe the extra singularity at  $z = \pm 1$ , we firstly determine the coefficients of the polynomials  $\alpha_e(z)$  and  $\alpha_o(z)$  using the cross section in the off-forward region  $-z_1 \leq z \leq z_1$ . Notation  $\sigma_{smooth}(z)$  will be used for the cross section which is obtained by using the polynomials thus determined, because this is the smooth continuation of the cross section from the off-forward region to whole the physical region  $-1 \leq z \leq 1$ . When the nuclear forces are short range, the observed cross section must coincide with

$\sigma_{smooth}(z)$  in the forward region  $z_1 \leq z \leq 1$  within the error. On the other hand, if the long range force exists and the extra singularity is of the form  $(1-z)^\gamma$ , then the observed cross section deviates from  $\sigma_{smooth}(z)$  in the forward region. We can evaluate such a deviation if the spectral functions of the long range force Eqs.(11) and (12) are given.

By using the amplitude of the long range force  $W(\nu, z)$  introduced in Eq.(14),  $\alpha(z)$ 's are written as

$$\alpha_e(z) = (W(\nu, z) + W(\nu, -z) - 2w_0 - 2w_2z^2) + c'_0 + c'_2z^2 \quad (21)$$

and

$$\alpha_o(z) = (W(\nu, z) - W(\nu, -z) - 2w_1z) + c'_1z \quad , \quad (22)$$

in which  $w_i$ 's are chosen in such a way that  $(w_0 + w_1z + w_2z^2)$  becomes the best fit to  $W(\nu, z)$  in the given off-forward region  $-z_1 \leq z \leq z_1$ . In particular for  $z_1 = 0$ ,  $w_0$ ,  $w_1$  and  $w_2$  are the value, slope and curvature at  $z = 0$  respectively. With these  $\alpha(z)$ 's we can compute the cross section  $\sigma_{long}(z)$  from Eq.(15). The relative deviation of the cross section  $\Delta\sigma/\sigma$  is defined by

$$\Delta\sigma/\sigma(\theta) = \frac{\sigma_{long}(z) - \sigma_{smooth}(z)}{\sigma_{smooth}(z)} \quad (23)$$

and if we retain only the first order of the small quantities  $\alpha_e(z)$ ,  $\alpha_o(z)$  and  $\Delta$  in Eq.(15), the relative deviation reduces to

$$\begin{aligned} \Delta\sigma/\sigma(\theta) &= \tilde{g}^s(\theta)(W(\nu, z) + W(\nu, -z) - 2w_0 - 2w_2z^2) + \\ &+ \tilde{g}^t(\theta)(W(\nu, z) - W(\nu, -z) - 2w_1z) \quad , \end{aligned} \quad (24)$$

where

$$\tilde{g}^s(\theta) = \frac{1}{2\sigma_0(\theta)} \text{Re}(\tilde{f}^s(\theta) + 2 \exp[i\delta_0^E] \sin \delta_0^E) \quad (25)$$

and

$$\tilde{g}^t(\theta) = \frac{3}{2\sigma_0(\theta)} \text{Re}(\tilde{f}^t(\theta) + 6\delta_{1c}z) \quad . \quad (26)$$

In Eqs.(25) and (26),  $\sigma_0(\theta)$  is the zeroth order of the cross section, and obtained by setting the samall quantities  $\alpha_e(z)$ ,  $\alpha_o(z)$  and  $\Delta$  in Eq.(15) equal to zero.

In figure 3 and 4, the coefficient functions  $\tilde{g}^s(\theta)$  and  $\tilde{g}^t(\theta)$  are displayed, in which we use the phase shifts  $\delta_0^E = 50.96^\circ$  and  $\delta_{1c} = 0.516^\circ$  at  $T_{lab} = 20$  MeV. , and also the  $\pi$ -N coupling constant  $g^2/4\pi = 14.4$  where  $g^2/4\pi = 4m^2f^2$ .

In figure 5, the relative deviation  $\Delta\sigma/\sigma(\theta)$  is plotted against  $\theta$ , the scattering angle in the center of mass system. The numbers attached to the curves are  $\theta_1$  the boundary of the off-forward region  $\theta_1 \leq \theta \leq 180^\circ - \theta_1$ , in which the parameters in  $\alpha(z)$ 's are determined. The curves show narrow dips at  $\theta = 15^\circ$  with the depth around 1 per cent. This is our prediction of the interference pattern derived from the parameters of the long range force. If the long range force does not exist, the curves of the relative deviation  $\Delta\sigma/\sigma(\theta)$  must be zero within the error.

In figure 6, the curves  $\Delta\sigma/\sigma(\theta)$ , in which the F-wave as well as the P-wave term in  $\alpha_o(z)$  are retained, are shown. The dips appear at  $\theta = 13^\circ$ .



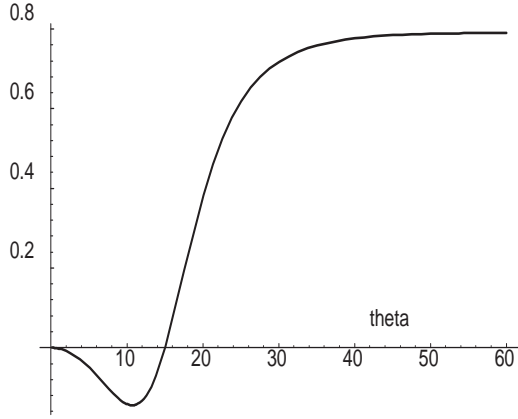


Figure 3: The singlet coefficient function  $\tilde{g}^s(\theta)$  is plotted against  $\theta$ .  $T_{lab} = 20$  MeV.

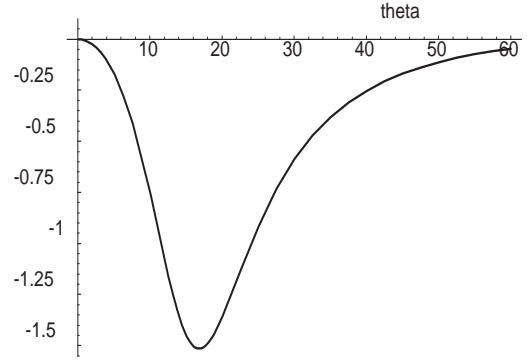


Figure 4: The triplet coefficient function  $\tilde{g}^t(\theta)$  is plotted against  $\theta$ .  $T_{lab} = 20$  MeV.

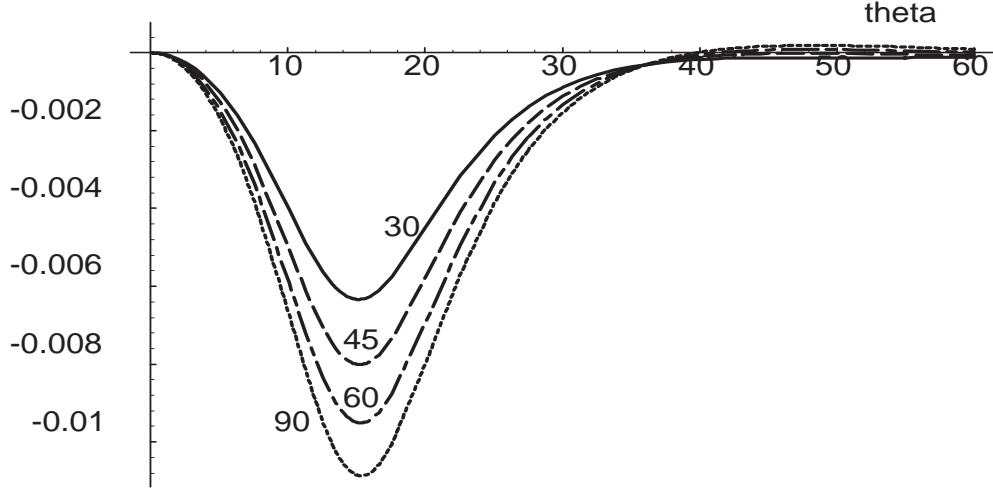


Figure 5: The relative deviation  $\Delta\sigma/\sigma(\theta)$  is plotted against  $\theta$  for  $T_{lab} = 20$  MeV.. Three parameters are determined in the off-forward region  $\theta_1 \leq \theta \leq 180^\circ - \theta_1$ . The curves correspond to  $\theta_1 = 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$  respectively.

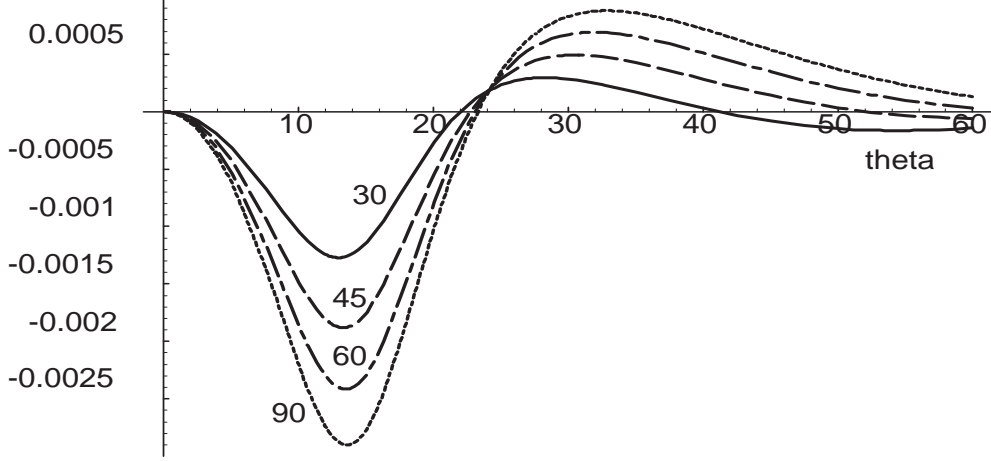


Figure 6: The relative deviation  $\Delta\sigma/\sigma(\theta)$  is plotted against  $\theta$  for  $T_{lab} = 20$  MeV.. Four parameters are determined in the off-forward region  $\theta_1 \leq \theta \leq 180^\circ - \theta_1$ . The curves correspond to  $\theta_1 = 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$  respectively.

## 4 Remarks and Comments

In this paper we propose to measure precisely the angular distribution of the cross section of the low energy proton-proton scattering, in order to confirm the long range interaction in the nuclear force. By using the parameters of the spectrum of the long range force obtained from the analysis of the S-wave phase shift of the p-p scattering, we predict the characteristic interference pattern in the angular distribution of the p-p cross section, which has a dip at  $\theta_{c.m.} = 14^\circ$  with the depth around 1 %. The observation of such a pattern is the more direct way to confirm the long range force, because the energy dependent phase shifts are obtained from the data of different laboratories by constructing a consistent curve, in which the correction factor of the incident beam is assigned to each experiment. If we consider the precision of the measurement the low energy proton-proton is an ideal place to observe the strong Van der Waals force.

Another good place to observe the Van der Waals force is the low energy ( $T_{lab} \sim 1$  MeV.) neutron-Pb scattering.[7] This is because the strength of the long range potential is magnified by a factor A, the mass number, and it is relatively easy to observe the anomaly of the angular distribution. Since the Van der Waals force is universal, we can expect to observe such a force also in other processes such as in the  $\pi$ - $\pi$  scattering.[8] Although the precisions of the  $\pi$ - $\pi$  data are not very high, the P-wave amplitude of the  $\pi$ - $\pi$  is another place easy to observe the strong Van der Waals force, because we can compute the spectral function of the two-pion exchange from the  $\pi$ - $\pi$  data, and by removing the spectrum from the amplitude we can prepare the wide domain of analyticity. If all the forces are short range, the Kantor amplitude must be almost constant.[9]

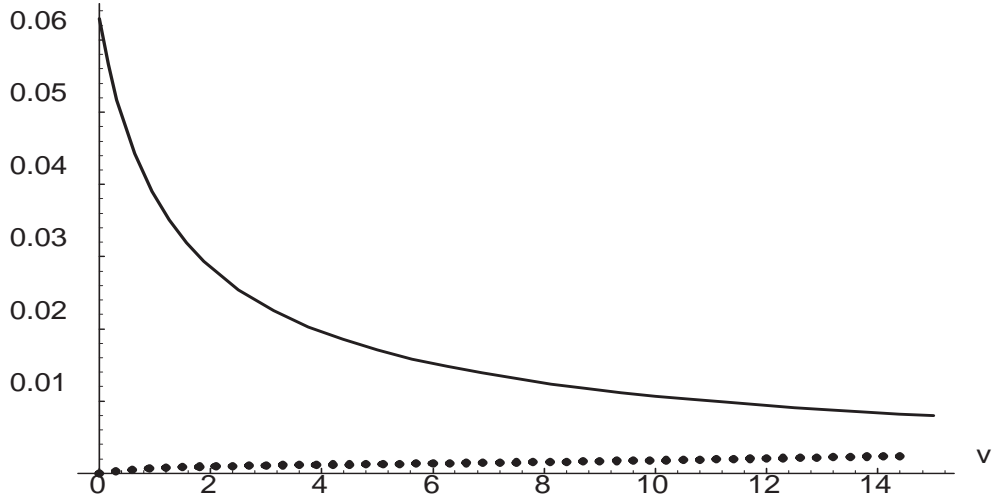


Figure 7: The once subtracted Kantor amplitude of the P-wave of the  $\pi$ -  $\pi$  scattering  $K_1(\nu)/\nu$  is plotted against  $\nu$ . The dotted curve is  $-K_1^{2\pi}(\nu)/\nu$ , the two-pion exchange contribution. If all the forces are short range, two curves must coincide up to a constant.

In figure 7, the once subtracted Kantor amplitude  $K_1(\nu)/\nu$  of the P-wave pi-pi scattering is shown along with the contribution from the two-pion exchange spectrum. Here again the cusp of the attractive sign appears. By making the chi-square fit, the parameters are determined.

$$\gamma = 1.95 \quad , \quad C' = 0.0161 \quad \text{and} \quad \beta = 0.144 \quad (27)$$

The long range force in  $\pi$ - $\pi$  is close to the Van der Waals force of the Casimir-Polder type rather than the London type.

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